A SIMULATION OF GPS AND DIFFERENTIAL GPS SENSORS

by

Dr. James M. Rankin, P.E.
Electrical Engineering Department
St. Cloud State University
St. Cloud, MN 56301

Introduction

The Global Positioning System (GPS) is a revolutionary advance in navigation. Users can determine latitude, longitude, and altitude by receiving range information from at least four satellites. The statistical accuracy of the user's position is directly proportional to the statistical accuracy of the range measurement. Range errors are caused by clock errors, ephemeris errors, atmospheric delays, multipath errors and receiver noise. Selective Availability which the military uses to intentionally degrade accuracy for non-authorized users is a major error source. The proportionality constant relating position errors to range errors is the Dilution of Precision (DOP) which is a function of the satellite geometry.

Receivers separated by relatively short distances have the same satellite and atmospheric errors. Differential GPS (DGPS) removes these errors by transmitting pseudorange corrections from a fixed receiver to a mobile receiver. The corrected pseudorange at the moving receiver is now corrupted only by errors from the receiver clock, multipath, and measurement noise.

This paper describes a software package that models position errors for various GPS and DGPS systems. The error model is used in the Real-Time Simulator and Cockpit Technology workstation simulations at the NASA-Langley Research Center. The GPS/DGPS sensor can simulate enroute navigation, instrument approaches, or on-airport navigation.

Pseudorange Errors

Pseudorange, a noisy estimate of range, is a measurement of the travel time for the satellite signals. The Course/Acquisition (C/A) code and Precision (P) code are the satellite signals used for the pseudorange measurement. The pseudorange, ρ model

$$\rho = r + \delta_{eph} + \delta_{iono} + \delta_{trop} + \delta_{SA} + cT + \delta_{mp} + \nu_{rcvr}$$

includes actual range (r) plus ephemeris (δ_{eph}), ionosphere (δ_{iono}), troposphere (δ_{trop}), SA (δ_{SA}), clock (cT), multipath (δ_{mp}), and measurement (ν_{rcvr}) errors. A measurement of the carrier phase can also be used to determine range. The carrier phase measurement (in meters) is

$$\lambda\phi = r - d_{iono} + d_{trop} + \delta_{SV} + \delta_{SA} + \lambda N + \delta_{mp} + \nu_{carrier}$$

The carrier phase measurement has an integer ambiguity, λN, that reflects uncertainty about the exact number of wavelengths traveled. By maintaining carrier lock, the value of N can be determined. (Ionosphere delay is negative for phase and positive for code.)

The ephemeris, ionosphere, troposphere, SA, and multipath errors are modeled as Gauss-Markov, correlated noise processes. These processes have an exponential autocorrelation function, $R(\tau) = \sigma^2 e^{-\beta|\tau|}$. The standard deviation and time constants are listed in Table 1. Gauss-Markov terms are simulated by the difference equation

$$x_{k+1} = e^{-\beta * \Delta T} x_k + w_k$$

where x is the parameter being simulated and w is Gaussian white noise.

Measurement noise is modeled as Gaussian white noise. Code correlators track to 1% of its signal width (1σ). Carrier phase measurements are accurate to 1% of its wavelength. Measurement noise parameters are shown in Table 2.

Receiver Models

Pseudorange accuracy is dependent on the satellite signals measured and the receiver sophistication. Since the error sources are independent of each other, the pseudorange error is the sum of errors from each noise source . The pseudorange autocorrelation for a stand-alone C/A code receiver is shown with SA off (in Figure 1) and SA on (in Figure 2). When SA is on, using more sophisticated receivers will not reduce errors significantly.

A DGPS system removes the SA and atmospheric errors that are common in receivers separated by less than 20 km. The remaining errors are due to multipath and measurement noise. The differentially corrected code and carrier phase measurements are

$$\overline{p} = r + cT + (\delta_{mp} + v_{code})_{rcvr} - (\delta_{mp} + v_{code})_{base}$$

$$\lambda\phi = r + cT + (\delta_{mp} + v_{carrier})_{mobile} - (\delta_{mp} + v_{carrier})_{base} + \lambda N$$

Dilution of Precision (DOP)

DOP values are calculated from the direction cosine vectors pointing from the receiver to the SATs. Statistical error on the X-axis is described as σ_x= XDOP * s_p where σ_p is the pseudorange error. YDOP and VDOP correspond to the Y and Z axes. HDOP is the horizontal DOP formed by combining X and Y into a radial 2D error. DOP values for Denver Stapleton airport are shown in Table 3. The values were calculated from the model every 10 seconds for one day.

Software Implementation

The GPS error model is written in C. gps_init_() initializes the data parameters used in the GPS simulation. gps_() is a 1 Hz routine that provides XYZ errors. The user can select from nine GPS/DGPS modes and enable/disable SA. The calcdop_() routine calculates XDOP,YDOP, and VDOP. A 24 SAT constellation is simulated using circular orbits and a spherical earth. The DOP values are determined using an all-in-view strategy with an elevation mask of 5 degrees above the horizon. Figure 3 shows the GPS error algorithm.

The horizontal accuracy at the 2σ (95%) level is shown in Table 4 for the GPS and DGPS systems modeled. The table assumes an HDOP of 1.6.

References

1) R. Grover Brown and Patrick Y.C. Hwang. "Introduction to Random Signals and Applied Kalman Filtering, 2nd ed." John Wiley & Sons. New York, NY, 1992.
2) Technical Characteristics of the NAVSTAR GPS, North Atlantic Treaty Organization (NATO) Navstar GPS Technical Support Group. June 1991.
3) Kremer, G; Kalafus, R; Loomis, P; and Reynolds, J. "The Effect of Selective Availability on Differential GPS Corrections". Navigation. Vol. 37, Spring 1990.
4) Cannon, M.E., and LaChapelle, G. "Analysis of a High-Performance C/A-Code GPS Receiver in Kinematic Mode". Navigation, Vol. 39, No. 3, Fall 1992.

Error parameter	Standard Dev. (1σ, meters)	Time Constant (1/ß, seconds)
Ephemeris	3	1800
Ionosphere	5	1800
Troposphere	2	3600
Multipath, C/A	5	600
Multipath, P	1	600
Multipath, L1 carrier	0.048	600
Integer ambiguity	1	1000
Selective Availability	30	180

Table 1. Gauss-Markov parameters for correlated noise sources.

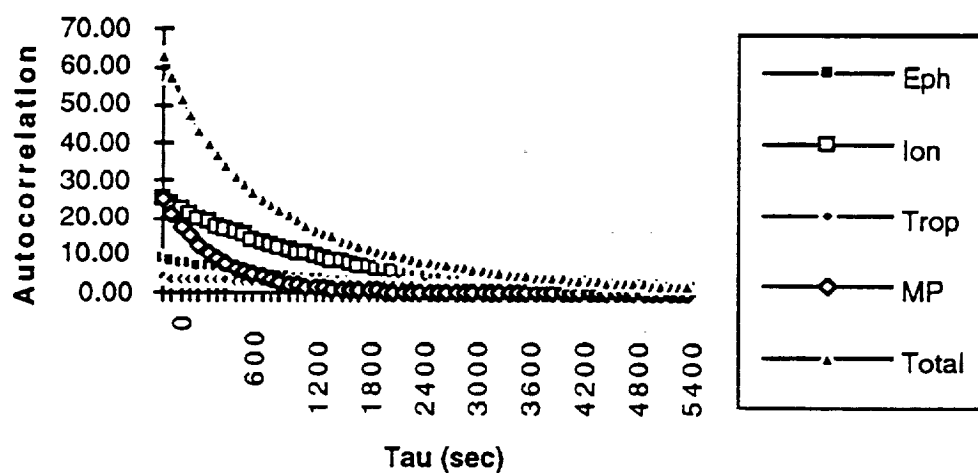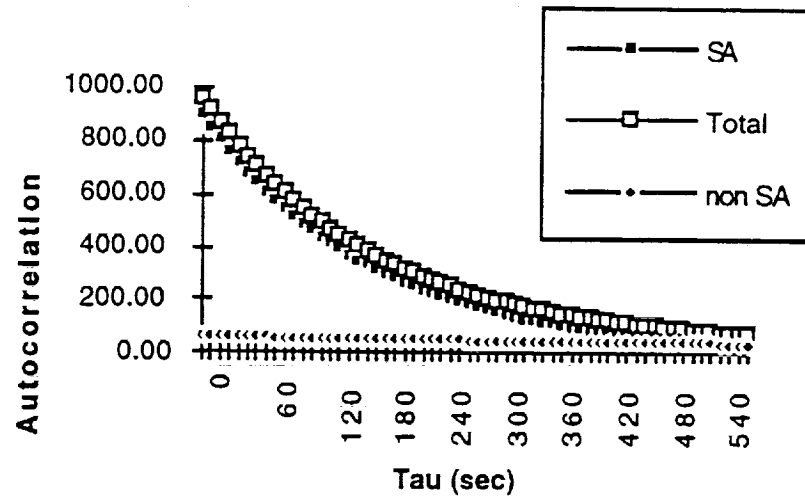| Model	Standard Deviation (meters)
C/A code, standard corr.	3.0
C/A code, narrow corr.	0.1
P code	0.3
L1 Carrier	0.0019

Table 2. Measurement noise statistics.

Figure 1. Autocorrelation of C/A code noise parameters (except Selective Availability)

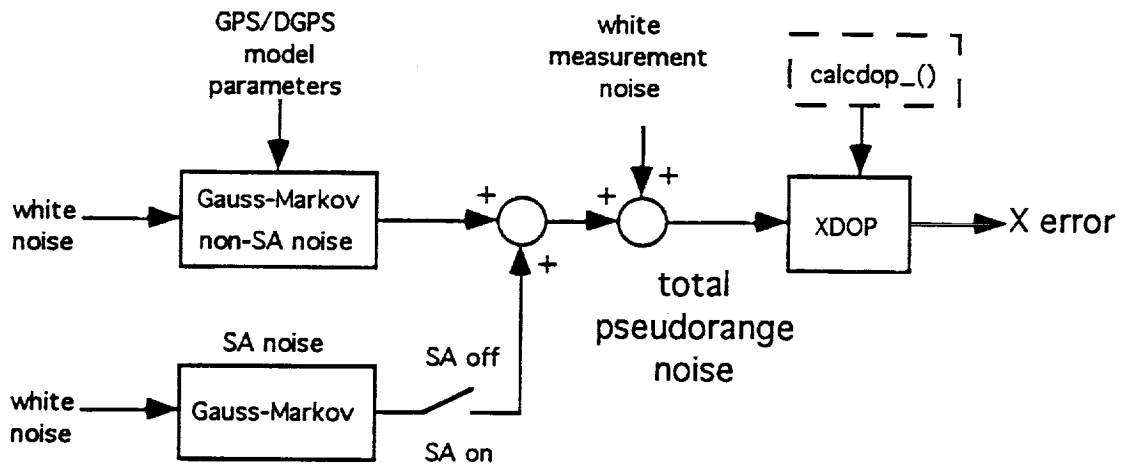Figure 2. Autocorrelation of C/A code noise including Selective Availability

Figure 3. GPS error algorithm for X-axis. (Y and Z axis are similar)

	Average	Minimum	Maximum
XDOP	0.7	0.5	0.9
YDOP	0.8	0.5	1.6
VDOP	1.1	0.7	1.8
HDOP	1.6	1.0	2.9
GDOP	2.1	1.4	4.1

Table 3. DOP values at Denver Stapleton airport. Calculated every ten seconds for one day.

GPS/DGPS RECEIVERS	SA off	SA on
GPS - C/A code wide correlator	27.2	99.8
GPS - C/A code narrow correlator	19.7	98.0
GPS - C/A code & carrier phase	13.9	97.0
GPS - P code	12.0	96.7
DGPS - C/A code wide correlator	-	18.7
DGPS - C/A code narrow correlator	-	1.2
DGPS - P code	-	3.4
DGPS - C/A code & carrier phase	-	0.2

Table 4. Horizontal accuracy for GPS/DGPS receivers (95%) (HDOP=1.6)